Name:

SM3 7.2: Graphing Cosecant & Secant

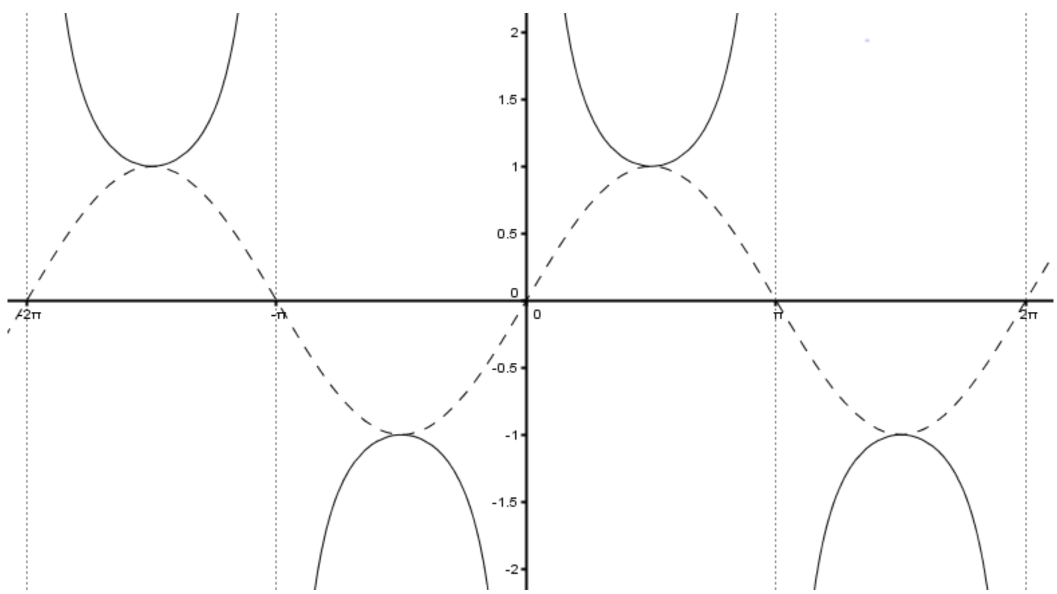
Vocabulary: period, asymptote

Cosecant Notes: Since the cosecant function is the reciprocal of the sine function, then we know a few things about the cosecant function.

x	$sin\ sin$	$csc\ csc\ x$
0	0	Undefined
$\frac{\pi}{2}$	1	1
π	0	Undefined
$\frac{3\pi}{2}$	-1	-1

Whenever $sin\ sin\ x\ = 1$, then $csc\ csc\ x$ also equals 1. And at the zeros of the sine function, the cosecant function has asymptotes. Since the period of the sine function is 2π, then the period of the cosecant function is also 2π. Local maximums on the sine function correspond to local minimums on the cosecant function, and local minimums on the sine function correspond to local maximums on the cosecant function.

Below is the graph of cosecant function, along with the graph of $y = sin\ sin\ x$.



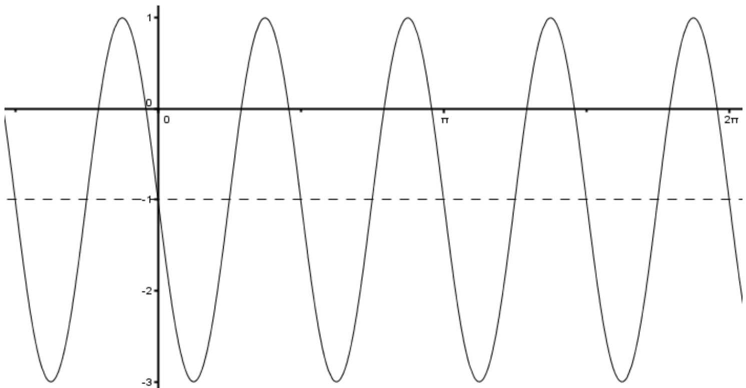
Extended form of cosecant function: $y = a\ csc\ csc\ \left(b(x-h)\right) + k$

Period of the cosecant function: $\frac{2\pi}{b}$

Example: Sketch the graph of $f(x) = 2\ csc\ csc\ (4x - 3\pi)\ - 1$.

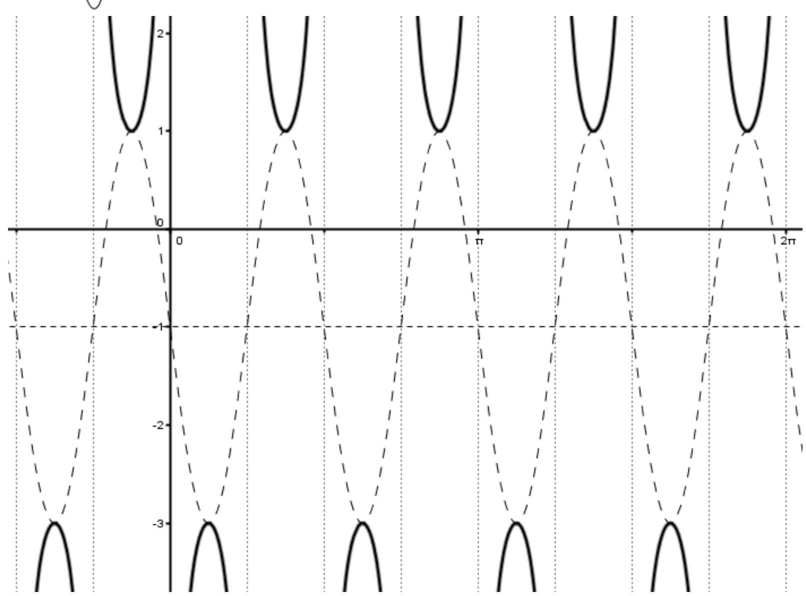
Note that $k = -1$, $a = 2$, and $b = 4$. To find the value of h we need to factor the inside of the cosecant function. $4x - 3\pi = 4\left(x - \frac{3\pi}{4}\right)$ So $h = \frac{3\pi}{4}$.

Consider what those values would do the sine function. $k = -1$ shifts it down, $a = 2$ increases the amplitude to 2 (max at $y = 1$, min at $y = -3$), $b = 4$ changes the period to $\frac{2\pi}{4} = \frac{\pi}{2}$ and $h = \frac{3\pi}{4}$ shifts it right $\frac{3\pi}{4}$.



Where the sine function crosses the midline, is where the asymptotes of the cosecant function occur. And the maximums $y = 1$ become minimums for the cosecant function. And the minimums $y = -3$ become maximums for the cosecant function.

Put it all together to get the cosecant graph.

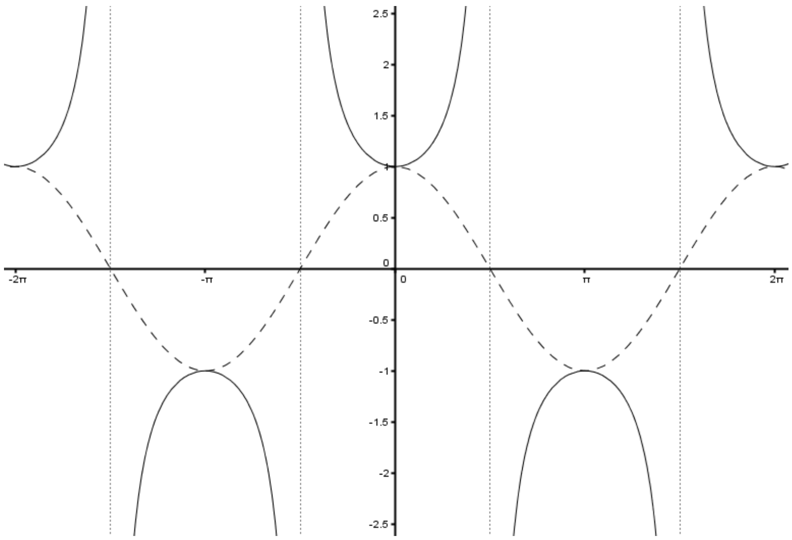


Secant Notes: Since the secant function is the reciprocal of the cosine function then we can determine a few things about it also.

x	$cos\ cos$	$sec\ sec\ x$
0	1	1
$\frac{\pi}{2}$	0	Undefined
π	-1	-1
$\frac{3\pi}{2}$	0	Undefined
2π	1	1

Whenever $cos\ cos\ x\ = \pm 1$, then $sec\ sec\ x\ = \pm 1$ also. And wherever $cos\ cos\ x\ = 0$, the secant function will have an asymptote. And the period of the secant function is 2π because that is the period of its reciprocal.

Below is the graph of the secant function, along with the cosine function.



Extended form of secant function:

$$y = a\ sec\ sec\ \left(b(x-h)\right) + k$$

Period of secant function: $\frac{2\pi}{b}$

Problems:

Identify the period and relative max/min values of each function.

1) $y = csc\ csc\ (3x)$

2) $f(x) = 4\ csc\ csc\ (x)$

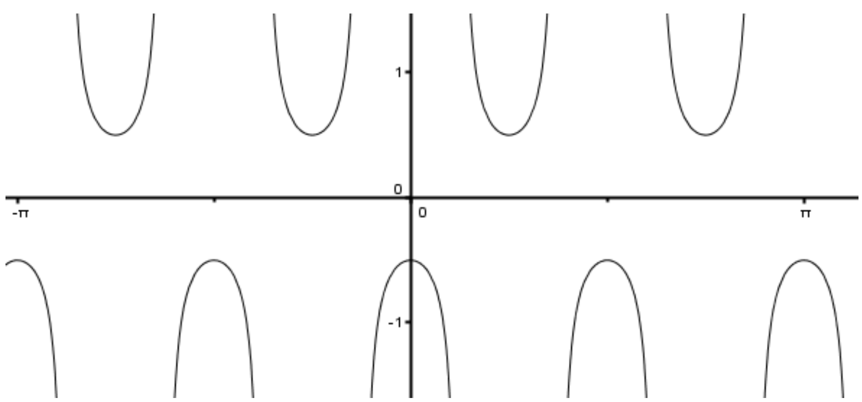
3) $g(x) = 1 + sec\ sec\ (x)$

4) $y = 3\ sec\ sec\ \left(\frac{1}{2}x\right)$

5) $y = .5\ csc\ csc\ \left(x - \frac{\pi}{2}\right)$

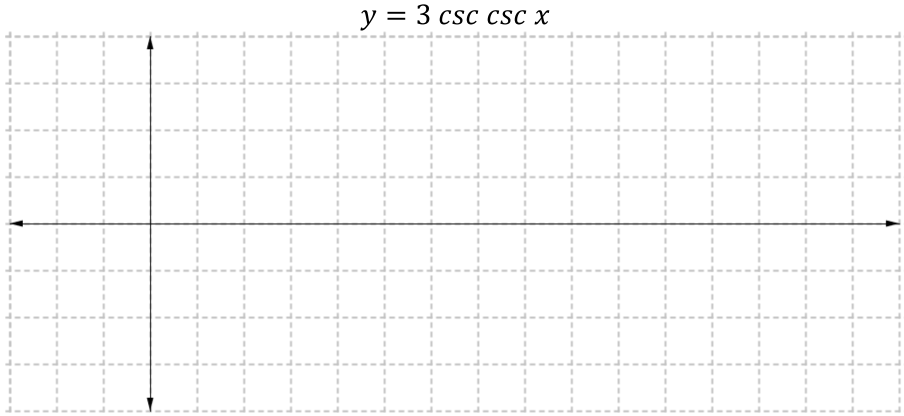
6) $h(x) = \left(2x - \frac{3\pi}{4}\right) - 3$

7)



Sketch an appropriate coordinate axis and graph two periods of the function.

8) $y = 3\ csc\ csc\ x$



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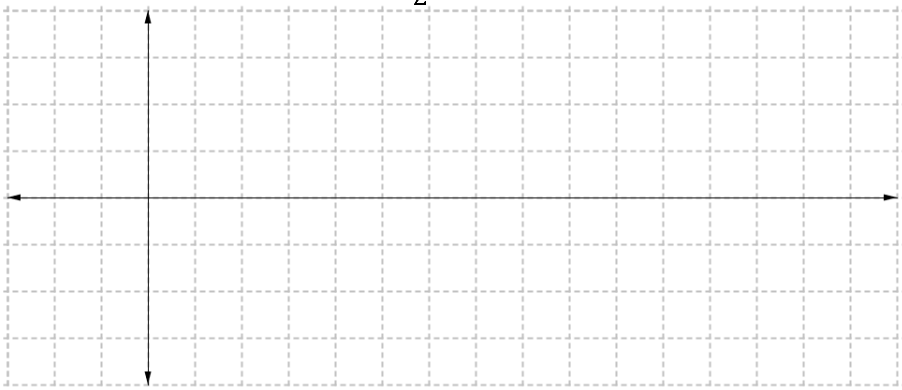
9) $y = \frac{1}{2} sec\ sec\ x$



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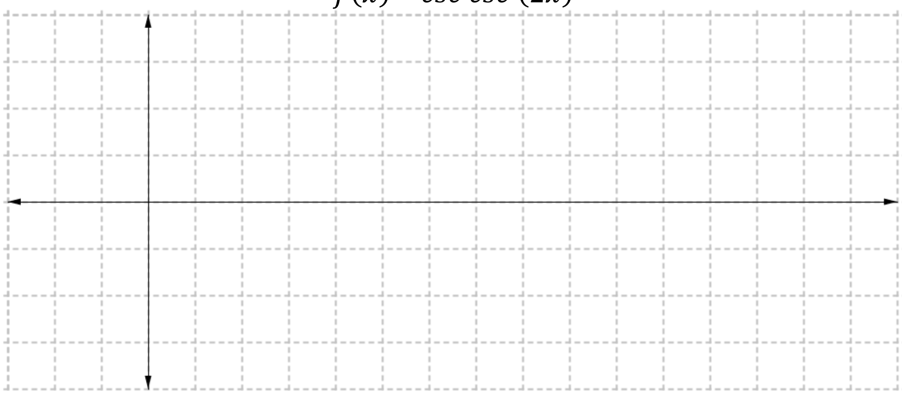
10) $f(x) = csc\ csc\ (2x)$



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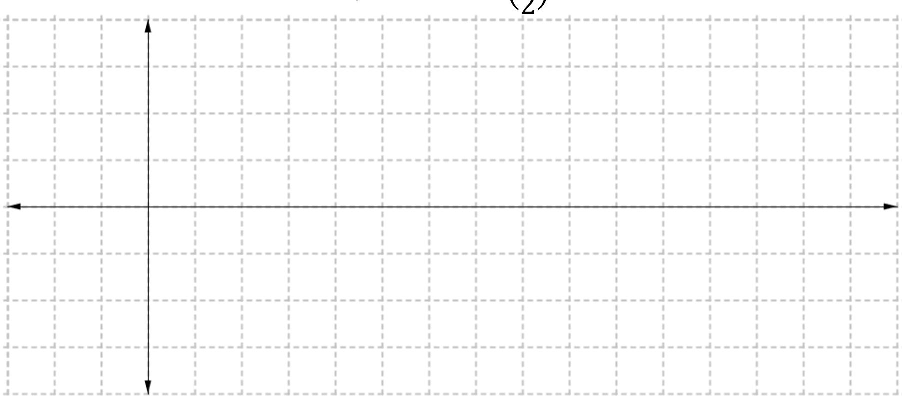
11) $y = sec\ sec\ \left(\frac{x}{2}\right)$



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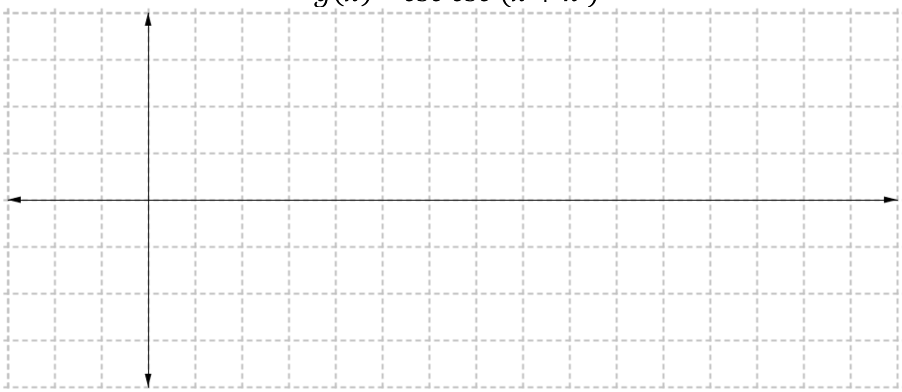
12) $g(x) = csc\ csc\ (x + \pi\ )$



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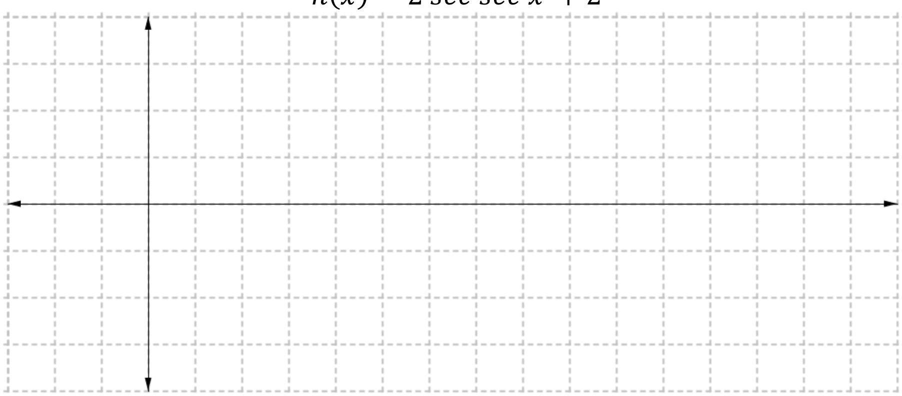
13) $h(x) = 2\ sec\ sec\ x\ + 2$



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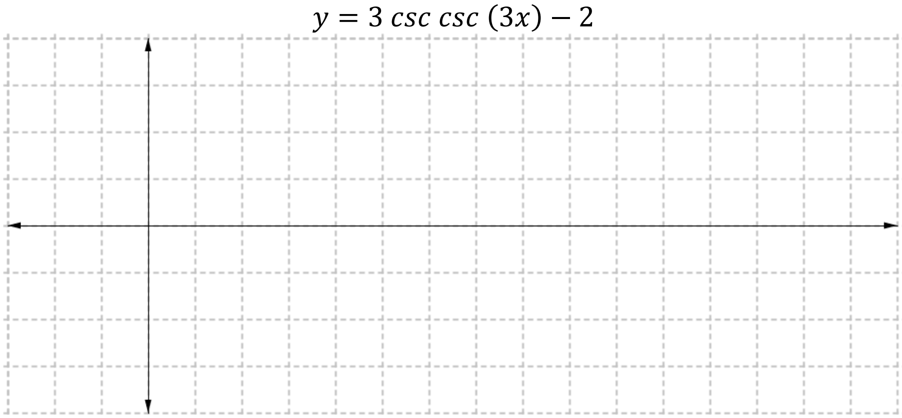
14) $y = 3\ csc\ csc\ (3x) - 2$



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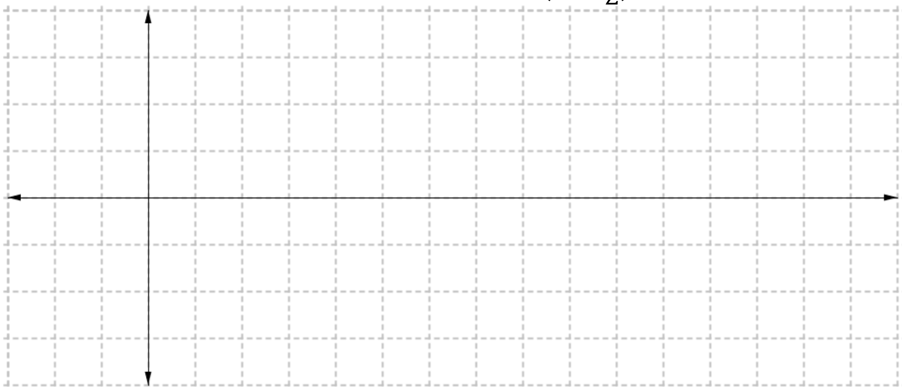
15) $f(x) = -1 + sec\ sec\ \left(x - \frac{\pi}{2}\right)$



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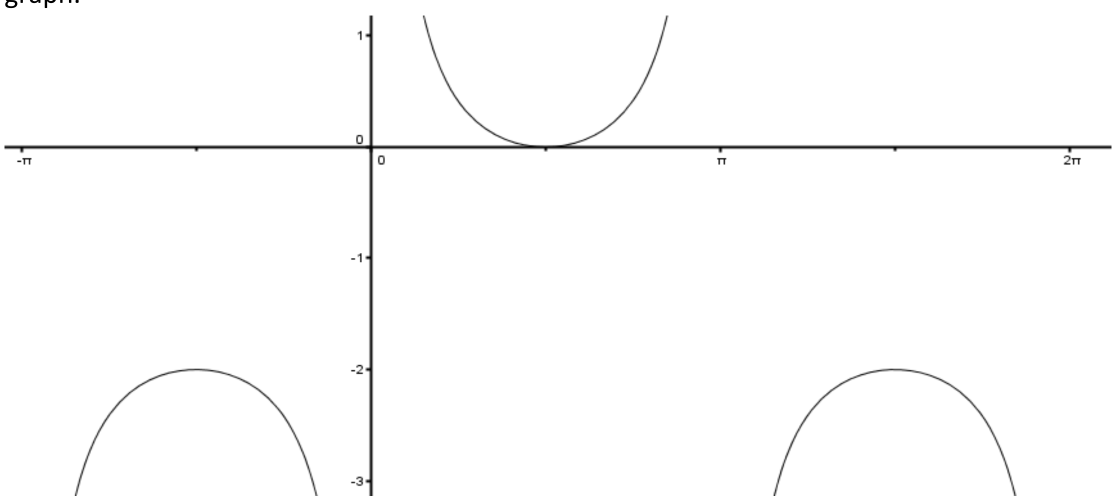
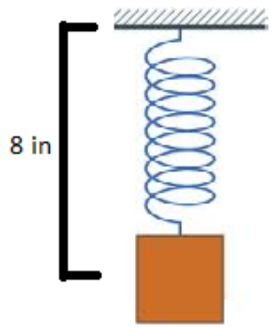
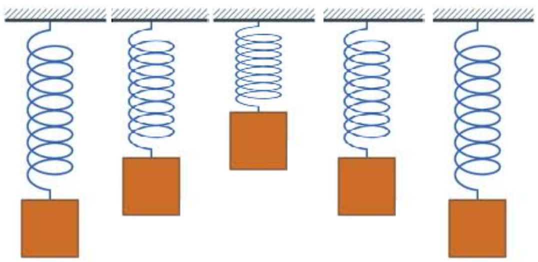
16) Write the simplest form of a) the cosecant function and b) the secant function for the given graph.





A spring with a weight attached to the end of the spring is hanging from a ledge. While at rest, the distance from the ledge to the middle of the weight is 8 inches.

Ian grabs the weight and tugs down on it, displacing it another 4 inches from the ledge and then (at time = 0) releases the spring so that it begins oscillating (it moves from being over-extended, to being at rest, to being under-extended, to being at rest, to being over-extended, etc.). It takes 4 seconds to complete the cycle of motion.



The diagram to the left shows snapshots of the weight's movement. Note that the spring is at rest in the second and fourth graphic.

17) Fill out the displacement values in the table, indicating how far the weight is from the ledge t seconds after Ian releases the weight (displacement below an object should be negative):

Time (t):	0	1	2	3	4	5	6	7	8	9
Displacement:										

18) What is the amplitude of the motion of the weight (include units)?

19) As the weight moves, it follows the points of a trigonometric curve. The picture above shows the weight's first five positions along the curve. Which trigonometric function does the weight's displacement seem to follow: $sin\ sin\ x$, $-\ sin\ sin\ x$,$cos\ cos\ x$, $or\ -cos\ cos\ x$?

20) How long is the period the weight takes to complete one cycle of motion (include units)?

21) Use the formula $Period = \frac{2\pi}{b}$ to determine an appropriate value for b.

22) How far is the weight from the ledge when the spring is at rest?

23) Write a trigonometric function that accurately describes the displacement of the weight as a function of time.